EEE401F: Digital Signal Processing
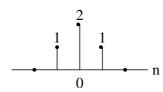
Class Test 1

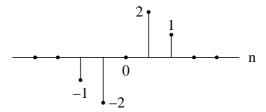### 7 April 2005

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.

- This test has *four* questions, totalling 20 marks.

- Answer *all* the questions.

- You have 45 minutes.

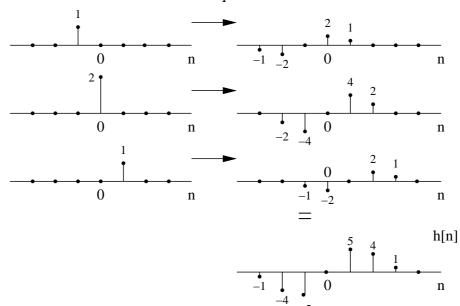1. (5 marks) A system has an impulse response

What is the output of the system when the input is the following:

Is the system causal? Is it stable? Why?

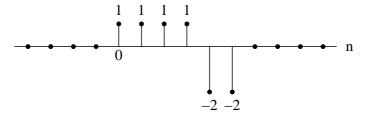The output is the convolution of the two sequences.

$h[n]$

The impulse response is nonzero for some negative values of n, so the system is not causal.

The sum

$$S = \sum_{n=-\infty}^{\infty} |h[n]| = 1 + 2 + 1 = 4 < \infty,$$

so the system is stable.

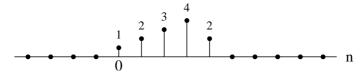2. (5 marks) The impulse response of an LTI system is shown below:

Determine and sketch the response of this system to the input $x[n] = u[n-4]$.
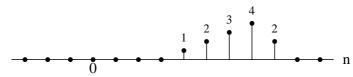
The response of the system to the unit step is

$$g[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k],$$

since $u[n-k]$ equals unity for $k \leq n$ and is otherwise zero. Thus the step response is just the accumulated impulse response:

The response to $x(t) = u(t-4)$ is just the response to the unit step, shifted by 4 units to the right:

3. (5 marks) A highpass FIR filter is characterised by the following impulse response:

$$h[n] = \begin{cases} -0.03 & n = 0 \\ -0.43 & n = 1 \\ 0.56 & n = 2 \\ -0.43 & n = 3 \\ -0.03 & n = 4 \\ 0 & \text{otherwise} \end{cases}$$

Write down the coefficients of an equivalent lowpass filter using frequency translation.

Frequency translation can be represented by the following transform pair:

$$e^{j\omega_0 n} x[n] \xleftrightarrow{\mathcal{F}} X(e^{j(\omega-\omega_0)}),$$

so multiplication by a complex exponential corresponds to shift in frequency.

If the coefficients describe a highpass filter, then the filter must be passing frequencies at $\omega = \pm\pi, \pm 3\pi, \ldots$, and blocking frequencies $\omega = 0, \pm 2\pi, \ldots$. If we shift this response by $\omega_0 = \pi$, then we will get a lowpass filter passing $\omega = 0, \pm 2\pi, \ldots$ and blocking $\omega = \pm\pi, \pm 3\pi, \ldots$.

The impulse response of the corresponding lowpass filter will be

$$h_{\text{LPF}}[n] = e^{j\pi n} h_{\text{HPF}}[n],$$

so

$$h_{\text{LPF}}[n] = \begin{cases} -0.03 & n = 0 \\ 0.43 & n = 1 \\ 0.56 & n = 2 \\ 0.43 & n = 3 \\ -0.03 & n = 4 \\ 0 & \text{otherwise} \end{cases}$$

4. (5 marks) An LTI system has the following system function:

$$H(z) = \frac{z^{-32}}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}$$

(a) Sketch the corresponding pole-zero diagram.

(b) List every possible region of convergence (ROC) for H(z).

(c) For each ROC, comment on system causality and stability.

(d) One of the ROCs should correspond to a causal system. In that case only, find the impulse response of the system.