EEE401F: Digital Signal Processing

Class Test 1

7 April 2005

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 20 marks.
- Answer *all* the questions.
- You have 45 minutes.

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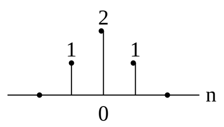
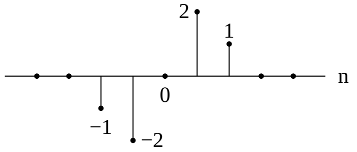
1. (5 marks) A system has an impulse response

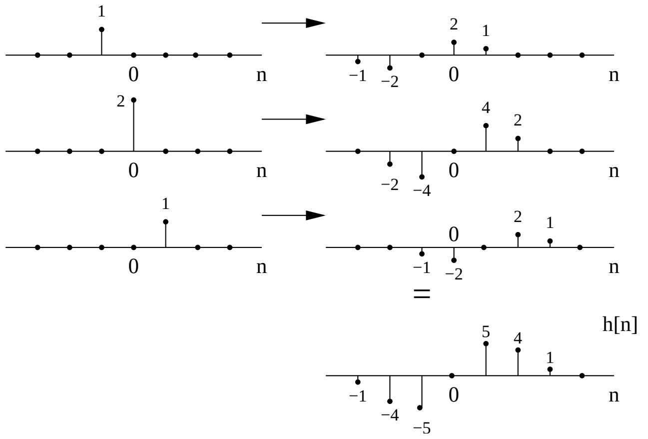


What is the output of the system when the input is the following:



Is the system causal? Is it stable? Why?

The output is the convolution of the two sequences.

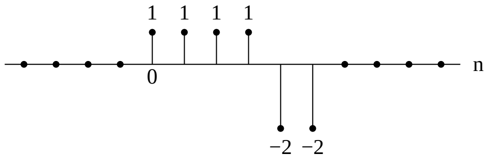


The impulse response is nonzero for some negative values of n, so the system is not causal. The sum

$$S = \sum_{n=-\infty}^{\infty} |h[n]| = 1 + 2 + 1 = 4 < \infty,$$

so the system is stable.

2. (5 marks) The impulse response of an LTI system is shown below:

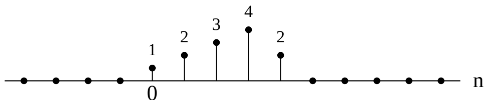


Determine and sketch the response of this system to the input $x[n] = u[n-4]$.

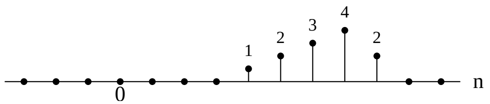
The response of the system to the unit step is

$$g[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k],$$

since $u[n-k]$ equals unity for $k \leq n$ and is otherwise zero. Thus the step response is just the accumulated impulse response:



The response to $x(t) = u(t-4)$ is just the response to the unit step, shifted by 4 units to the right:



3. (5 marks) A highpass FIR filter is characterised by the following impulse response:

$$h[n] = \begin{cases} -0.03 & n = 0 \\ -0.43 & n = 1 \\ 0.56 & n = 2 \\ -0.43 & n = 3 \\ -0.03 & n = 4 \\ 0 & \text{otherwise} \end{cases}$$

Write down the coefficients of an equivalent lowpass filter using frequency translation.

Frequency translation can be represented by the following transform pair:

$$e^{j\omega_0 n}x[n] \stackrel{\mathcal{F}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)}),$$

so multiplication by a complex exponential corresponds to shift in frequency.

If the coefficients describe a highpass filter, then the filter must be passing frequencies at $\omega = \pm\pi, \pm 3\pi, \ldots$, and blocking frequencies $\omega = 0, \pm 2\pi, \ldots$. If we shift this response by $\omega_0 = \pi$, then we will get a lowpass filter passing $\omega = 0, \pm 2\pi, \ldots$ and blocking $\omega = \pm\pi, \pm 3\pi, \ldots$.

The impulse response of the corresponding lowpass filter will be

$$h_{\text{LPF}}[n] = e^{j\pi n} h_{\text{HPF}}[n],$$

so

$$h_{\text{LPF}}[n] = \begin{cases} -0.03 & n = 0 \\ 0.43 & n = 1 \\ 0.56 & n = 2 \\ 0.43 & n = 3 \\ -0.03 & n = 4 \\ 0 & \text{otherwise} \end{cases}$$

4. (5 marks) An LTI system has the following system function:

$$H(z) = \frac{z^{-32}}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}$$

(a) Sketch the corresponding pole-zero diagram.

(b) List every possible region of convergence (ROC) for H(z).

(c) For each ROC, comment on system causality and stability.

(d) One of the ROCs should correspond to a causal system. In that case only, find the impulse response of the system.