EEE401F EXAM
DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2005
3 hours

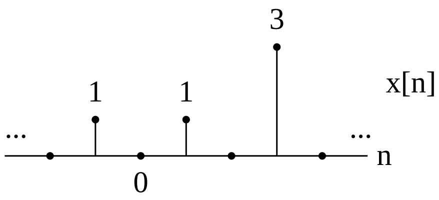
Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *seven* questions totalling 85 marks. You must answer all of them.
- **Part B** has *three* questions, each counting 15 marks. You must answer *one* of them.
- A table of standard z-transform pairs appears at the end of this paper.
- A formula sheet for the radar/sonar question appears at the end of this paper.
- You have 3 hours.

PART A

Answer all of the following questions.

1. If $x[n]$ is the signal



then plot the signals

(a) $1 - 2x[n]$

(b) $-2x[2n]$

(c) $-2x[-n] + 2$

(d) $x[n]\delta[n-1]$

(e) $x[-n]u[2+n]$

(10 marks)

2. Consider the two sequences

$$x_1[n] = \begin{cases} 4 & n = 0 \\ 2 & n = 1 \\ 5 & n = 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad x_2[n] = \begin{cases} 2 & n = 0 \\ 1 & n = 1 \\ 3 & n = 2 \\ 6 & n = 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Convolve the sequences using the Z-transform.

(b) *Explain in detail* how you would convolve the sequences using the FFT.

(10 marks)

3. Sketch the magnitude and phase response of the filter

$$H(z) = \frac{\prod_{k=1}^{3}(z - e^{j2\pi k/4})}{z^3}.$$

What is the ROC of the z-transform?

(10 marks)

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4. Evaluate the magnitude and phase of the frequency response of the following recursion at π rad/s, where $x[n]$ is the input and $y[n]$ the output:

$$y[n] = 0.7(y[n-1] + x[n-1]) - x[n-3].$$

The sampling rate is 2 samples per second.

(15 marks)

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5. Find the impulse response of the system

$$F(z) = \frac{3z^2 - 2}{(z-1)(z-0.5)}$$

for region of convergence $0.5 < |z| < 1$. Is the system stable? Is the system causal? Give a LCCDE recursion for the system.

(15 marks)

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6. The four-point DFTs of two discrete-time signals are

$$X[k] = [22, -4 + j2, -6, -4 - j2]$$
$$Y[k] = [8, -2 - j2, 0, -2 + j2].$$

If $w[n]$ is the 4-point circular convolution of $x[n]$ with $y[n]$ (that is, $w[n] = x[n] \textcircled{N} y[n]$ for N=4), find $w[2]$.

(10 marks)

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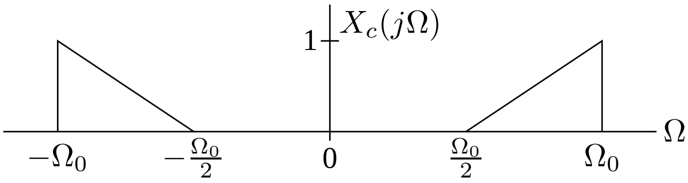
7. A continuous-time signal $x_c(t)$, with Fourier transform



is sampled with sampling period $T = 2\pi/\Omega_0$ to form the sequence $x[n] = x_c(nT)$.

(a) Sketch the Fourier transform $X(e^{j\omega})$ for $|\omega| < \pi$.

(b) The signal $x[n]$ is to be transmitted across a digital channel. At the receiver, the original signal $x_c(t)$ must be recovered. Draw a block diagram of the recovery system and specify its characteristics. Assume that ideal filters are available.

(c) In terms of Ω_0, for what range of values of T can $x_c(t)$ be recovered from $x[n]$?

(15 marks)

PART B

Answer *one* of the following three questions. Each question counts 15 marks.

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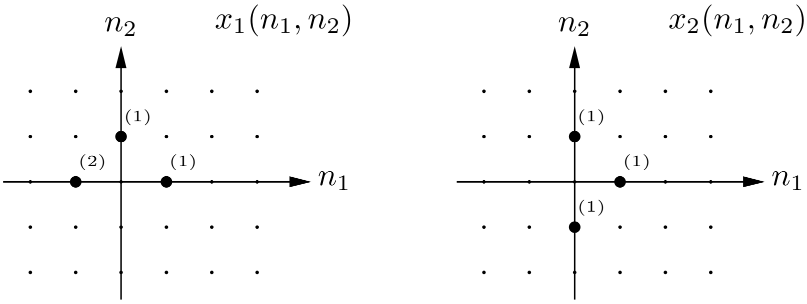
1. **Multidimensional signal and image processing**

   (a) Find the 2-D convolution of the following two signals:



(5 marks)

   (b) Describe how you would go about detecting vertical edges (like the edges of buildings) in an image. Describe the role of 2-D convolution (or filtering) in the process, and discuss the choice of filter parameters. How would you reduce the effects of noise in the edge detection process?

(5 marks)

   (c) Explain why a periodic sequence in 2-D is more complex than for 1-D.

(5 marks)

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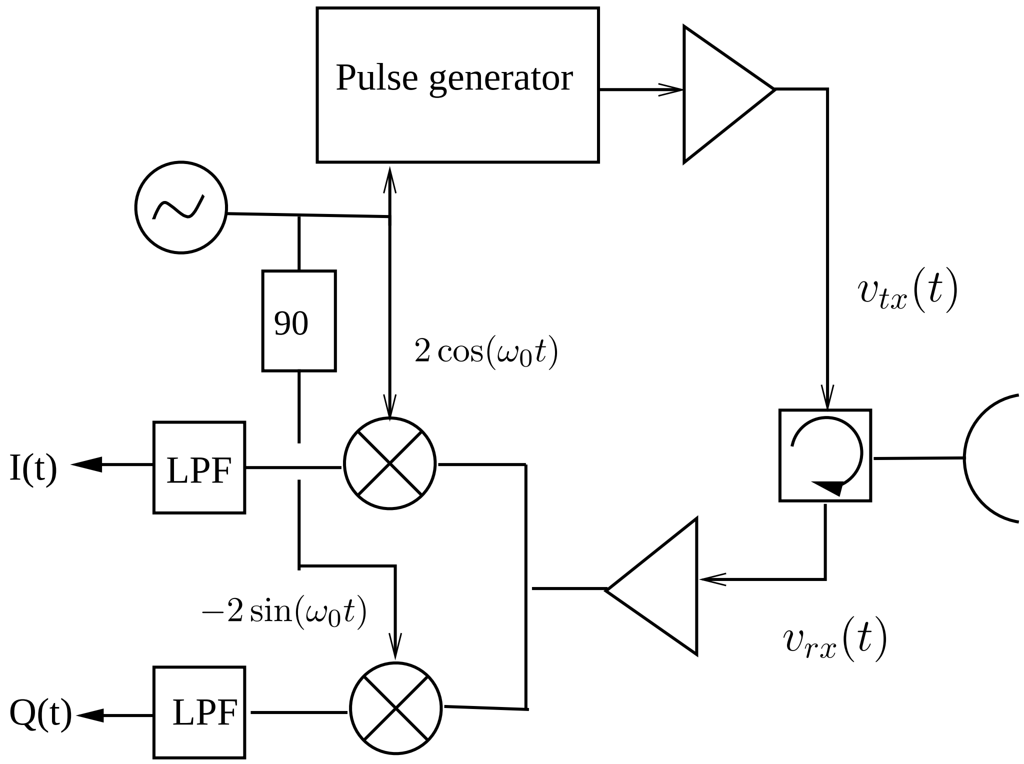
2. **Radar/sonar signal processing**

A simplified block diagram of a radar is shown below:



(a) Draw a neatly labelled block diagram of an equivalent analytic signal model of the radar.

(2 marks)

(b) Illustrate with the aid of sketches of the *frequency spectra*, how the signals in the system are related, particularly illustrate the relationship between the *frequency* spectra of the following signals:

i. the impulse response of the scene $\xi(t) \leftrightarrow \xi(f)$

ii. the transmitted rf pulse $v_{tx}(t) \leftrightarrow V_{tx}(f)$ and baseband form $p(t) \leftrightarrow P(f)$

iii. complex baseband signal $v_{bb}(t) \leftrightarrow V_{bb}(f)$.

(3 marks)

(c) A digital signal processing algorithm must be developed for pulse compression and display of the echoes received by the radar. The transmitted pulse is a *chirp* pulse with bandwidth of 50 MHz, and a centre frequency of 12 GHz. The processor operates on the baseband signals sampled from the IQ down-converter.

i. What sample rate is required for the signals at the output of the IQ down-converter?

ii. What digital signal processing steps would you carry out to obtain a processed “range profile” of the scene, considering that you would like to optimize *the signal to noise* *ratio*?

iii. Calculate the 3dB range resolution in metres.

(5 marks)

(d) If a *deconvolution* filter is used to process the radar data in (c), sketch the point target response (both magnitude and phase) as a function of range, that you would expect to see at the output for a point target at range of 1000m.

(3 marks)

(e) What additional processing step can one implement to improve the sidelobe levels of the point target response? Sketch a typical output and indicate clearly the effect of such processing, compared to the output of the deconvolution filter in (d).

(2 marks)

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3. **General**

A causal function $y[n]$ has Z-transform

$$Y(z) = \frac{z^3}{z^3 - 3z^2 + 5z - 9}.$$

(a) Find the Z-transform of $y_1[n] = y[n-3]u[n-3]$.

(b) Evaluate $y[n]$ for $n = 0$ and $n = 3$ by expanding the appropriate Z-transform into power series.

(15 marks)

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$